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Renormalization group flows from $D = 3$, $N = 2$ matter coupled gauged supergravities

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Abstract: We study holographic RG flows of $N = 2$ matter coupled AdS_3 supergravities which admit both compact and non-compact sigma manifolds. For the compact case the supersymmetric domain wall solution interpolates between a conformal IR region and flat spacetime and this corresponds to a deformation of the CFT by an irrelevant operator. When it is non-compact, the solution can be interpreted as a flow between an UV fixed point and a non-conformal(singular) IR region. This is an exact example of a deformation flow when the singularity is physical. We also find a non-supersymmetric deformation flow when the scalar potential has a second AdS vacua. The ratio of the central charges is rational for certain values of the size of the sigma model. Next, we analyze the spectrum of a massless scalar on our background by transforming the problem into Schrödinger form. The spectrum is continuous for the compact case, yet it can be both continuous (with or without mass gap) and discrete otherwise. Finally, 2-point functions are computed for two examples whose quantum mechanical potentials are of Calogero type.

Keywords: AdS-CFT and dS-CFT Correspondence, Supergravity Models.

Contents

1. Introduction

After the celebrated AdS/CFT conjecture [1]–[3] renormalization group (RG) flows from gauged supergravities have been studied extensively (see [4]–[6] for review and references). However, $D = 3$ seems to be an exception although it has the obvious advantage of being dual to a 2-dimensional CFT. To the best of our knowledge only papers where RG flows are worked in detail using 3D-supergravities are [7, 8] in which an exact RG flow from $N = 8$ model with $\mathrm{SO}(4) \times \mathrm{SO}(4)$ gauge symmetry was found and its correlation functions were analyzed.

One reason for this neglect might be that, matter coupled AdS_3 supergravities are not as familiar as their higher dimensional relatives and their construction is still in progress. Therefore, we would like to begin with a brief summary of the current status in $D = 3$. The $N = 2$ supergravity coupled to an arbitrary number of scalar supermultiplets was constructed in [9, 10]. In [10] scalars are charged under the $\mathrm{U}(1)$ R-symmetry group and consequently they have a potential, whereas in [9] there is only a cosmological constant. The connection between these two models for the flat sigma model was described in [11]. In [10] the $N = 1$ truncation was obtained too. Another $N = 1$ model with a different sigma-model manifold was given in [12]. The boundary symmetries of [10] were studied in [11] and its extension by including a Fayet-Iliopoulos term was given in [13]. The maximal ($N = 16$) gauged supergravity was constructed in [14, 15] and its vacua were studied in [16, 17]. Recently a topological generalization of $N = 16$ model was obtained in [18]. The $N = 8$ matter coupled AdS_3 supergravities were constructed in [19]. There also has been some effort to obtain AdS_3 gauged supergravities directly by dimensional reduction [20]–[22].

The $N = 2$ model that we will study here [10] has the virtue of being simple and wealthy at the same time. It admits both compact and non-compact sigma model manifolds and it has three free parameters: Gravitational coupling constant, cosmological constant and the size of the sigma manifold which we denote by "a". This extra freedom gives a richness since the value of "a" changes the shape of the potential drastically when the scalar manifold is non-compact.

The organization of this paper is as follows; in the next section 2 we will review the $N = 2$ gauged supergravity and rederive the supersymmetric domain wall solution obtained in [10] in a form which is more useful for our aims. In general, there is an ambiguity whether a solution is a true deformation or a different vacua of the same theory [23, 24]. Our first goal is to resolve this for our model. When the sigma manifold is compact the superpotential is a trigonometric function of the scalar field. Such a superpotential was studied in [25] for $D = 5$, $N = 2$ gauged supergravity and our result is similar too: The supersymmetric solution interpolates between a conformal IR region and flat spacetime. This is created by deforming the dual CFT by an addition of an irrelevant operator. In the non-compact case, the solution is asymptotically AdS and it exhibits a naked singularity. We will show that this can be interpreted as a RG flow to a non-conformal IR theory [26]. When $a^2 \leq 1/2$ this corresponds to a non-renormalizable scalar mode and therefore to a deformation of the CFT by an addition of a relevant operator. Whereas, for $a^2 > 1/2$ the flow is driven by giving a nonzero expectation value to the operator. The singularity is unphysical for this case since the potential becomes unbounded [27] in supersymmetric solution. When $1/2 < a^2 < 1$ the potential has another (stable) AdS extrema and one may try to construct a non-supersymmetric interpolating solution. However, an exact solution is hard to obtain. Instead we will follow the analysis done in [28] for $D = 7$, $N = 1$ gauged supergravity and find a numerical solution. This actually is enough to conclude that interpolating solution is a true deformation. We also show that the ratio of the central charges and operator dimensions are rational for certain values of a^2. In section 3, we solve the wave equation for a minimally coupled scalar in our background by casting the problem into Schrödinger form. The spectrum is continuous for the compact sigma manifold, while it can be continuous (there is a mass gap for $a^2 = 1/4$) or discrete otherwise, depending on a^2. Moreover, we compute the 2-point functions for two specific values of a^2 when sigma manifold is non-compact. The quantum mechanical potentials for these, belong to the class of potentials that appear in Calogero models (3-body problem in 1-dimension) as was first observed in [29] for $D = 5$, $N = 8$ gauged supergravity. We conclude in section 4 with some comments and possible future directions.

2. The matter coupled, $N = 2$ AdS_3 supergravity

The $N = 2\, AdS_3$ supergravity multiplet consists of a graviton $e_\mu{}^a$, a complex gravitini ψ_μ and a gauge field A_μ. The $N = 2$ scalar multiplet, on the other hand, contains n complex scalar fields ϕ^α and n complex fermions λ^r.

In [10], the sigma model manifold M was taken to be a coset space of the form G/H where G can be compact or non-compact and H is the maximal compact subgroup of G.

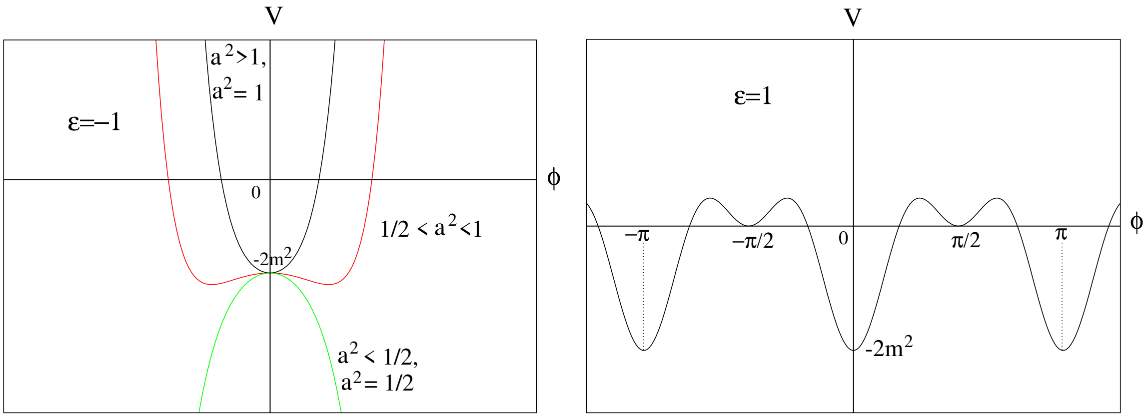

Figure 1: The potential $V(\phi)$ plotted against ϕ.

In particular, the following cases are considered:

$$M_+ = \frac{\mathrm{SU}(n+1)}{\mathrm{SU}(n)\times \mathrm{U}(1)}\,, \qquad M_- = \frac{\mathrm{SU}(n,1)}{\mathrm{SU}(n)\times \mathrm{U}(1)}\,. \tag{2.1}$$

Note that U(1) is the R-symmetry group. We define the parameter $\epsilon = \pm 1$ to indicate the manifolds $M_\pm$. In this paper we consider the cases $S^2 = \mathrm{SU}(2)/\,\mathrm{U}(1)$ and $H^2 = \mathrm{SU}(1,1)/\,\mathrm{U}(1)$, i.e, $n = 1$. For our purposes it is enough have a single real scalar, consequently we set $\phi = |\phi|$ and all other fields to zero.[1] To simplify the lagrangian given in [10] we make a redefinition for the scalar field as follows:

$$2\phi \equiv \begin{cases} \tanh\left(\dfrac{\hat\phi}{2}\right) & \epsilon = -1\,, \\ \tan\left(\dfrac{\hat\phi}{2}\right) & \epsilon = 1\,. \end{cases} \tag{2.2}$$

Then, we drop the hat for notational convenience, and the lagrangian of [10] becomes:

$$e^{-1}\mathcal{L} = \frac{1}{4}R - \frac{1}{4a^2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\,, \tag{2.3}$$

where the potential is given by (figure 1):

$$V = -2m^2\mathrm{C}^2\left[(1+2\epsilon a^2)\mathrm{C}^2 - 2\epsilon a^2\right]. \tag{2.4}$$

The function C is defined as:

$$\mathrm{C} = \begin{cases} \cosh\phi & \epsilon = -1\,, \\ \cos\phi & \epsilon = 1\,. \end{cases} \tag{2.5}$$

[1]This is consistent with the field equations given in [10]. The vector field equation implies that when the vector field is set to zero then the scalar field has to be real. The critical points of this truncated theory with real ϕ are also critical points of the full potential given in [10]. This has already been noted in [10] and can be seen from the form of potential which depends on the magnitude of the scalars at least quadratically.

The constant "a" is the characteristic curvature of $M_\pm$ (e.g. $2a$ is the inverse radius in the case of $M_+ = S^2$). The gravitational coupling constant κ has been set equal to one and $-2m^2$ is the AdS_3 cosmological constant. Unlike in a typical AdS supergravity coupled to matter, the constants κ, a, m are not related to each other for non-compact scalar manifolds, while a^2 is quantized in terms of κ in the compact case as $\kappa^2/a^2 = n$, where n is an integer [10].

The fermionic supersymmetry transformations of the model are:

$$\begin{aligned}
\delta\psi_\mu &= \left(\partial_\mu + \frac{1}{4}\omega_\mu{}^{ab}\gamma_{ab}\right)\varepsilon + \frac{1}{2}W\gamma_\mu\varepsilon\,, \\
\delta\lambda &= \frac{1}{2a}\left(-\gamma^\mu\partial_\mu\phi - a^2\frac{\partial W}{\partial\phi}\right)\varepsilon\,.
\end{aligned} \tag{2.6}$$

Here W is the superpotential and is given by:

$$W = -2\epsilon m \mathrm{C}^2\,. \tag{2.7}$$

This is defined up to an overall unimportant sign. The potential V can be written in terms of the superpotential as:

$$V = \frac{a^2}{4}\left(\frac{\partial W}{\partial\phi}\right)^2 - \frac{1}{2}W^2\,. \tag{2.8}$$

Let us now consider the following domain wall ansatz for the metric:

$$ds^2 = e^{2A(y)}(-dt^2 + dx^2) + dy^2\,. \tag{2.9}$$

The field equations of (2.3) with the above metric are:

$$\begin{aligned}
\phi'' + 2A'\phi' &= 2a^2\frac{\partial V}{\partial\phi} \\
A'' + 2(A')^2 &= -4V \\
2A'' + 2(A')^2 &= -\frac{(\phi')^2}{a^2} - 4V\,.
\end{aligned} \tag{2.10}$$

One of the three equations is redundant and can be derived from the other two. These are nonlinear equations and it is hard to find an exact solution except when there is supersymmetry. After imposing the vanishing of the supersymmetry conditions $\delta\lambda = 0$ and $\delta\psi = 0$ one obtains the following first order equations:

$$\phi' = a^2\frac{\partial W}{\partial\phi}\,, \tag{2.11}$$

$$A' = -W\,, \tag{2.12}$$

where the prime indicates differentiation with respect to y. In deriving these equations we also imposed the condition $\gamma^y\varepsilon = \epsilon\varepsilon$ which means that the solution is half-supersymmetry preserving. From $\delta\psi = 0$ the ϕ-dependence of the spinor ε is determined to be [10]:

$$\varepsilon = (\epsilon - \epsilon\mathrm{C}^2)^{1/8a^2}\,(1 - \epsilon\gamma_y)\varepsilon_0\,, \tag{2.13}$$

where ε_0 is an arbitrary constant spinor.

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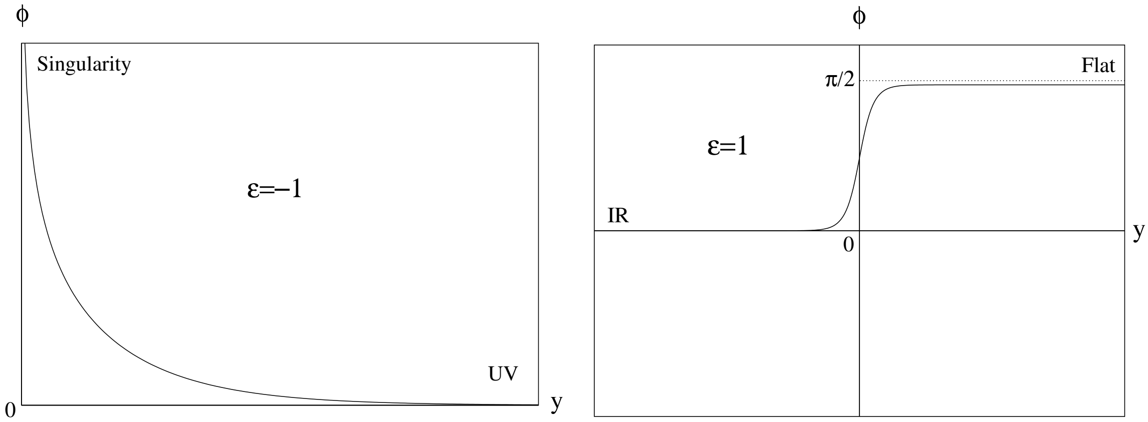

Figure 2: The superymmetric solution ϕ versus y graphs. For $\epsilon = -1$ there is a naked singularity at $y = 0$ ($\phi \to \infty$) and an UV fixed point at $y = \infty$ ($\phi \to 0$). For ϵ=1 there is an IR fixed point at $y = -\infty$ ($\phi \to 0$). The spacetime becomes flat as $y \to \infty$ ($\phi \to \pi/2$).

The equation (2.11) can be integrated easily and the result is (figure 2):[2]

$$\phi = \begin{cases} \dfrac{1}{2}\ln\left(\dfrac{1+e^{-4ma^2y}}{1-e^{-4ma^2y}}\right) & \epsilon = -1\,, \qquad 0 \le y < \infty \\ \arctan e^{4ma^2y} & \epsilon = 1\,, \qquad -\infty < y < \infty\,. \end{cases} \tag{2.14}$$

From this, A and W can be solved using (2.11) and (2.12):

$$\begin{aligned} A &= -\frac{\epsilon}{4a^2}\ln(e^{-8\epsilon ma^2y} + \epsilon)\,, \\ W &= \frac{2m}{1+\epsilon e^{8\epsilon ma^2y}}\,. \end{aligned} \tag{2.15}$$

Now $v = e^A$ is the RG scale of the CFT. If we denote the operator that is associated with the field ϕ on the boundary by $\mathcal{O}_\phi$, then its β-function is defined as [30]–[32]:

$$\beta = v\frac{d\phi}{dv} = -a^2\frac{1}{W}\frac{\partial W}{\partial \phi} = \begin{cases} -2a^2\tanh\phi & \epsilon = -1\,, \quad 0 \le \phi < \infty \\ 2a^2\tan\phi & \epsilon = 1\,, \quad 0 \le \phi \le \pi/2\,. \end{cases} \tag{2.16}$$

The sign of the derivative of the β-function determines the nature of the fixed point at $\phi = 0$. From (2.16) we deduce that $\phi = 0$ is an UV fixed point for $\epsilon = -1$ and an IR fixed point for $\epsilon = 1$.

Linearizing V around $\phi = 0$, one finds that mass of the field ϕ at the fixed point is:

$$M_\phi^2 = 16m^2a^2(a^2+\epsilon)\,. \tag{2.17}$$

[2]Here we made a coordinate change $y \to -y$ for $\epsilon = -1$ in order to be able to associate larger energies with increasing y. An integration constant y_0, which determines the location of the singularity, has been set to zero by using the translational invariance along the y direction.

From equations (2.10) we see that solution close to AdS boundary (ϕ=0) behaves as:[3]

$$\phi = \phi_0^+ e^{-(2-\Delta)y/R} + \phi_0^- e^{-\Delta y/R}\,. \tag{2.18}$$

Δ is the conformal dimension of the boundary operator $\mathcal{O}_\phi$ and $R = 1/(2m)$ is the AdS radius. For a relevant operator $\Delta < 2$. From $\Delta(\Delta-2) = M_\phi^2 R^2$ and (2.17) we get $\Delta = 1 + |1 + 2\epsilon a^2|$. Now, let us analyze $\epsilon = \pm 1$ cases separately.

2.1 Compact sigma manifold S^2 ($\epsilon = 1$)

The potential (2.5) has a minimum at $\phi = 0$ which is a supersymmetric AdS_3 vacuum and minima at $\cos\phi = 0$ corresponding to a supersymmetric 2+1 dimensional Minkowski vacua. The maxima at $\cos^2\phi = a^2/(2a^2+1)$ are non-supersymmetric de Sitter vacua (figure 1). At these extrema scalars have tachyonic mass of $-32m^2a^4(a^2+1)/(2a^2+1)$, so the de Sitter vacua are unstable. The superpotential (2.7) has two extrema which are located at $\cos\phi = 1$ (AdS vacua) and $\cos\phi = 0$ (Minkowski vacua). This type of superpotential was also studied in [25] for $N = 2, D = 5$ gauged supergravity. However, its results are general and apply to our case as well. As we mentioned following (2.16), $\phi = 0$ ($y = -\infty$) is an IR fixed point. The dimension of the operator $\mathcal{O}_\phi$ is $\Delta = (2 + 2a^2) \geq 2$, and around $y = -\infty$, from (2.14) we get $\phi \approx e^{-(2-\Delta)y/R}$ which means that we have a deformation of the CFT by an irrelevant operator [6].

2.2 Non-compact sigma manifold H^2 ($\epsilon = -1$)

The theory admits various critical points (figure 1):

(i) For $a^2 \leq 1/2$, the maximum at $\phi = 0$ is a supersymmetric AdS_3 vacuum.

(ii) For $1/2 < a^2 < 1$, $\phi = 0$ is a supersymmetric AdS_3 vacua and there are two minima at $\cosh^2\phi = a^2/(2a^2-1)$. Although these are non-supersymmetric AdS_3 vacua, they are stable in the sense that they satisfy the Breitenlohner-Freedman bound [33] ,i.e., $\partial^2 V/\partial\phi^2 = 16m^2a^2(1-a^2)/(2a^2-1) > -1$.

(iii) For $a^2 \geq 1$ the minimum at $\phi = 0$ is a supersymmetric AdS_3 vacuum.

Now, $y = \infty$ corresponds to an UV fixed point of the dual CFT. There is a naked singularity at $y = 0$ (figure 2).[4] This can be seen by looking at the metric (2.9) close to $y = 0$ using (2.15):

$$ds^2 = y^{1/(2a^2)}(-dt^2 + dx^2) + dy^2\,. \tag{2.19}$$

According to the analysis of [27] this singularity is unphysical when $a^2 > 1/2$ despite the supersymmetry.[5] The reason is that, for $a^2 > 1/2$ the potential becomes unbounded near the singularity at $\phi = \infty$ as can be seen from (figure 1). For $a^2 \leq 1/4$ the singularity

[3] In writing this equation we assume that $2a^2 \notin \mathbb{Z}$. Otherwise there are some subtleties. See [11] for a more detailed treatment.

[4] In figures 2–4 we assume, without loss of generality, $a^2 = 3/4$ and $m = 1$.

[5] Exactly the same interval was excluded in [11] since some fields become divergent on the boundary for $a^2 > 1/2$ and linear approximation applied in [11] fails.

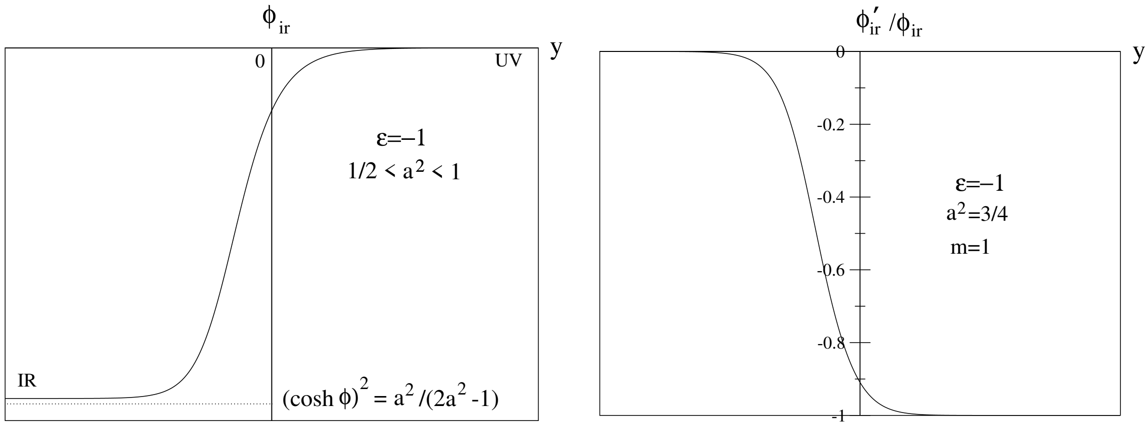

Figure 3: The non-supersymmetric solution $\phi_{\rm ir}$. This solution interpolates between two AdS vacua. $y = \infty$ is the UV and $y = -\infty$ is the IR region. From the second graph we see that $\phi_{\rm ir} \approx e^{-(2-\Delta)y/R} = e^{-2m(2-2a^2)}$ as $y \to \infty$.

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is null and as explained in more detail in [27] it is harder to understand the physics for $1/4 < a^2 \leq 1/2$ where the singularity is timelike.

In this supersymmetric solution the theory flows from an UV fixed point at $\phi = 0$ ($y = \infty$) to a non-conformal IR vacua at $\phi = \infty$. The rate which ϕ approaches to $y = \infty$ is e^{-4ma^2y} from (2.14). Since in the same limit the determinant of the metric (2.9) is $e^{2A} \approx e^{4my}$ from (2.15), we see that ϕ is square-integrable when $a^2 > 1/2$ and therefore corresponds to giving a nonzero vacuum expectation value to the operator $\mathcal{O}_\phi$. For $a^2 \leq 1/2$ the solution can be interpreted as deformation by a relevant ($a^2 = 0$ case is marjinal) operator. The form of the superpotential (2.7) resembles the one studied in [34] where their exact solution is associated with the RG flow from $N = 4$ super Yang-Mills theory to pure $N = 1$ in the IR by a deformation.

As we have discussed above, the supersymmetric solution does not physically make sense for $a^2 > 1/2$. However, for $1/2 < a^2 < 1$ one can look for a kink solution that interpolates between the maximum and a minimum[6] of the potential (figure 1). But, to do this one needs to solve the nonlinear field equations (2.10) with appropriate boundary conditions which is hard to do analytically. But, they can be solved numerically and the solution is given in (figure 3).[7] From the second graph in (figure 3) we see that, $\phi_{\rm ir}$ approaches to $y = \infty$ as $e^{-2m(2-2a^2)y}$ and it is not square-integrable. Thus, this solution describes a relevant deformation the UV lagrangian at $\phi = 0$ by adding the term $\int \phi\mathcal{O}_\phi$. The theory flows to a non-supersymmetric IR fixed point at $\phi = (1/2)\ln((1 - 2a\sqrt{1-a^2})/(2a^2 - 1))$ which is conformal again. At this extremum the reflection symmetry of the potential is broken and super-Higgs effect can be seen from (2.6).

Actually any solution of (2.11) and (2.12) with W satisfying the constraint (2.8) is a solution to second order equations [35, 36]. W's with this property depend on one

[6] Since the potential is symmetric around the y-axis it doesn't matter which minimum we choose. We will pick the one on the negative ϕ-axis for the forthcoming discussions.

[7] We follow the notation of [28] and use the subscript ir to indicate the numerical solution.

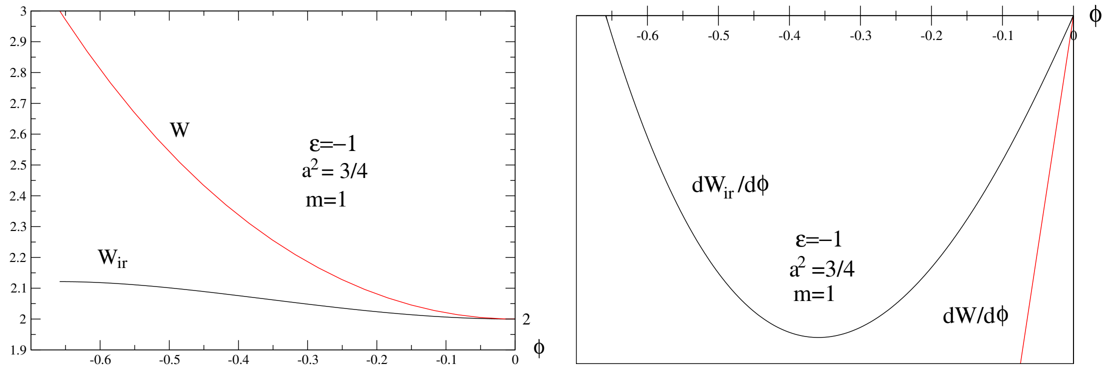

Figure 4: The comparison of the generating function $W_{\rm ir}$ and the true superpotential W. $\partial W_{\rm ir}/\partial\phi$ graph shows that $W_{\rm ir}$ has a second extremum located at $\phi = 1/2\ln((1-2a\sqrt{1-a^2})/(2a^2-1))$.

integration constant and only one of them is the true superpotential, the rest being the Hamilton-Jacobi generating functions [31]. Once the solution for $\phi_{\rm ir}$ is known (figure 3) the generating function $W_{\rm ir}$ can be constructed retrospectively [28] using equation (2.11) (figure 4). In order to compare $W_{\rm ir}$ and the true superpotential W we may make a Taylor series expansion[8] around $\phi = 0$ in (2.8). This is achieved by calculating the left-hand side of (2.8) from (2.4) and (2.5) and then comparing with the right-hand side term-by-term. Imposing $\partial W/\partial\phi = 0$ at $\phi = 0$ sets the coefficient of the term ϕ in W to zero and we obtain a second order equation for the coefficient of ϕ^2. The result is $W_{\rm ir} \approx 2m(1 + ((1-a^2)/a^2)\phi^2 + \cdots)$, whereas $W \approx 2m(1+\phi^2+\cdots)$. Even though W has only one extremum which is at $\phi = 0$, $W_{\rm ir}$ has two (figure 4).

The holographic $\mathcal{C}$-function can be defined as [38, 39, 32]:

$$\mathcal{C} = \frac{c_0}{A'}\,, \tag{2.20}$$

where, c_0 is a constant. Since equations (2.10) imply that $A'' < 0$, $\mathcal{C}$ decreases monotonically along the flow from UV to IR. This function should interpolate between the central charges $c_{\rm uv}$ and $c_{\rm ir}$. From (2.8) and (2.12) we see that at fixed points $A'^{\,2} = -2V$ and thus,

$$\frac{c_{\rm ir}}{c_{\rm uv}} = \frac{\sqrt{2a^2-1}}{a^2}\,, \qquad \frac{1}{2} < a^2 < 1\,. \tag{2.21}$$

Note that $c_{\rm ir}$ is less than $c_{\rm uv}$ as it should be by Zamolodchikov's $\mathcal{C}$-theorem [40]. The dimension of the operator at this extremum is $\Delta_{\rm ir} = 1+\sqrt{9-8a^2}$ from the potential (2.4). The ratio of the central charges is rational when $2a^2 = r^2+1$, where $r<1$ is a rational number. If, in addition there exists a rational number s such that $5 = 4r^2+s^2$, then operator dimensions in IR are rational too, as happened in [7].[9] For example, $a^2 = 5/8$ satisfies both conditions.

[8] This expansion is studied in detail in [37]. Our expansion agrees with theirs after appropriate scalings.
[9] For matter fermions, λ, the operator dimension is $\Delta_{\rm uv} = 1 + 1/2|1-4a^2|$ and $\Delta_{\rm ir} = 1 + (a^2/2)((|1-4a^2|)/(2a^2-1))$. [11]

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3. Spectrum of massless scalars

In this section we solve the Laplace equation for a massless scalar field Φ in our supersymmetric background (2.9). This problem has been investigated in higher dimensions first in [29] and [41]–[43]. We begin by making a coordinate change $dy = e^A dz$ and obtain a conformally flat metric:

$$ds^2 = e^{2A(z)}(-dt^2 + dx^2 + dz^2)\,. \tag{3.1}$$

Then, the $\Box_3\Phi = 0$ equation with the above metric becomes:[10]

$$(\partial_z^2 + \partial_z A\partial_z - \partial_t^2 + \partial_x^2)\Phi(t,x,z) = 0\,. \tag{3.2}$$

After making a separation of variable, $\Phi(t,x,z) = e^{i\vec{p}.\vec{x}}e^{-A/2}\Psi(z)$ we get,

$$\partial_z^2\Psi - V_{QM}\Psi = p^2\Psi\,, \tag{3.3}$$

where the potential is given by,

$$V_{QM} = \frac{1}{2}\partial_z^2 A + \frac{1}{4}(\partial_z A)^2\,. \tag{3.4}$$

Note that the potential can be written as $V_{QM} = \mathcal{U}(z)^2 + \mathcal{U}'(z)$ where the prepotential is $2\mathcal{U}(z) = \partial_z A$ and hence this is a supersymmetric quantum mechanics problem. This implies the positivity of the normalizable spectrum [45, 36, 46]. In other words, the physical spectrum is bounded from below, $-p^2 \geq 0$. It is usually enough to know the shape of V_{QM} to get some qualitative information about the dual CFT spectrum [41]. To accomplish this, it is sufficient to obtain coordinate z in terms of y near the end points from (2.15). For $\epsilon = -1$ the AdS boundary, i.e. $y = \infty$, is mapped into $z = z_0$ and the potential blows up quadratically,

$$V_{QM}(AdS) \approx \frac{3}{4(z-z_0)^2} \tag{3.5}$$

whereas the potential close to the singularity at $y = 0$ is

$$V_{QM}(\text{singularity}) \approx \frac{(3-8a^2)}{4(1-4a^2)^2 z^2}\,. \tag{3.6}$$

The singularity located at $y = 0$ is mapped into $z = 0$ and and $z = -\infty$ for $a^2 > 1/4$ and $a^2 \leq 1/4$ respectively. Therefore, for $a^2 \leq 1/4$ the potential approaches to a finite value and we have a continuous spectrum. Only $a^2 = 1/4$ case is unclear because of the denominator, which implies a mass gap as we will see below. When $a^2 > 1/4$ in order to have a discrete spectrum the coefficient of the potential should satisfy $(3-8a^2)/4(1-4a^2)^2 \geq -1/4$ from elementary quantum mechanics. This is indeed true for any a^2. The equality

[10] If we make a small perturbation of the metric (3.1) as $ds^2 = e^{2A}(\eta_{ij} + h_{ij})dx^i dx^j + e^{2A}dz^2$ it can be shown that the transverse, traceless part of h_{ij} obey the same wave equation (3.2) as happened in higher dimensions [44]. However, in 3 dimensions graviton has no degree of freedom and imposing $h_i^i = \partial^i h_{ij} = 0$ automatically implies $(-\partial_t^2 + \partial_x^2)h_{ij} = 0$. This suggests that scalar fluctuations should also be included to analyze metric fluctuations.

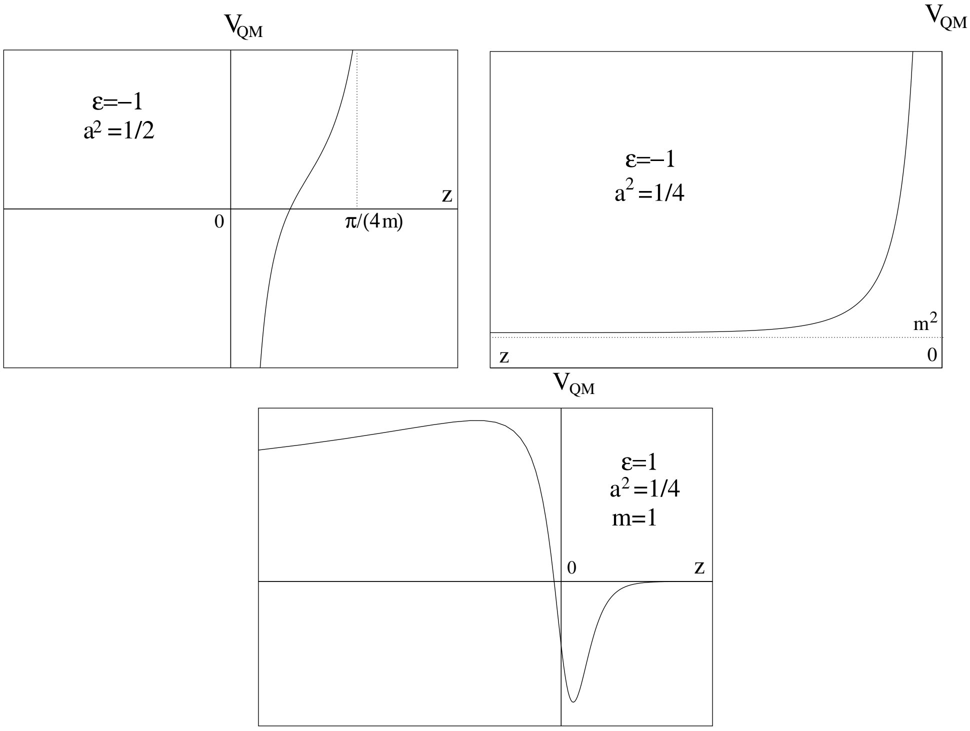

Figure 5: Quantum mechanical potentials. For $\epsilon = -1$ and $a^2 \leq 1/4$, $V_{QM} \to 0$ as $z \to -\infty$, except when $a^2 = 1/4$.

occurs at $a^2 = 1/2$. The potential can be solved exactly for $\epsilon = -1$ when $a^2 = 1/2$ and $a^2 = 1/4$:

$$V_{QM} = \begin{cases} \dfrac{4m^2(1 - 2\cos 4mz)}{\sin^2 4mz} & a^2 = \dfrac{1}{2}, \quad 0 \leq z < \dfrac{\pi}{4m} \\ \dfrac{2m^2 e^{2mz} + m^2}{(1 - e^{2mz})^2} & a^2 = \dfrac{1}{4}, \quad 0 < z < -\infty \end{cases} . \tag{3.7}$$

When $a^2 = 1/4$ we see that $V_{QM} \to m^2$ as $z \to -\infty$ which implies a mass gap (Figure 5). These potentials are among the ones studied in Calogero models as was noticed in [29] for AdS_5. The first one is a member of the Pöschl-Teller potentials (type I) and the second one belongs to Eckart type potentials. (See [47] for a review.)

For $\epsilon = 1$ the AdS boundary at $y = -\infty$ is mapped into $z = -\infty$ and the potential is given by (3.5). The flat region at $y = \infty$ is mapped into $z = \infty$ and $V_{QM} \to 0$ in this limit too. Therefore we have a continuous spectrum without any mass gap. Unfortunately we couldn't solve the potential in a closed form for any value of a^2. However, it can be drawn numerically; for instance, when $a^2 = 1/4$ it looks like in (figure 5).

3.1 2-point functions ($\epsilon = -1$)

We may check the above statements explicitly when $\epsilon = -1$ for $a^2 = 1/2$ and $a^2 = 1/4$ by calculating the 2-point functions. For $a^2 = 1/2$ let us define a new variable $u = \sin^2(z/R)$ where $R = 1/(2m)$. Now the singularity is at $u = 0$ and the AdS boundary is at $u = 1$.

The equation (3.3) becomes:

$$u(1-u)\Phi'' + (1-u)\Phi' - \frac{p^2R^2}{4}\Phi = 0\,. \tag{3.8}$$

This is a hypergeometric equation and its solution which is regular in the interior ($u = 0$) is:

$$\Phi = F\left(\sqrt{\frac{-p^2R^2}{4}}, -\sqrt{\frac{-p^2R^2}{4}}; 1; u\right). \tag{3.9}$$

The other linearly independent solution of (3.8) is logarithmically divergent at $u = 0$. Now, we can calculate the 2-point function using the prescription of [2, 48] and get:[11]

$$\langle\mathcal{O}(p)\mathcal{O}(-p)\rangle = -\frac{p^2R^2}{4}\left[\psi\left(1+\sqrt{\frac{-p^2R^2}{4}}\right) + \psi\left(1-\sqrt{\frac{-p^2R^2}{4}}\right)\right], \tag{3.10}$$

where $\psi = \Gamma'/\Gamma$ is the psi-function. The correlator has a discrete spectrum with poles located at $-p^2 = 4(n+1)^2/R^2$, $n = 0, 1, 2, \ldots$. Note that there is no zero-mass pole.

For $a^2 = 1/4$ we define a new variable $u = e^{z/R}$ and the equation (3.3) becomes:

$$u^2(1-u)\Phi'' + u(1-u)\Phi' - p^2R^2(1-u)\Phi = 0\,. \tag{3.11}$$

This is again a hypergeometric equation and its regular solution at $u = 0$ is:

$$\Phi = u^{-(1/2)+q}F\left(-\frac{1}{2}+q+\sqrt{p^2R^2}, -\frac{1}{2}+q-\sqrt{p^2R^2}; 1+2q; u\right), \qquad q = \frac{1}{2}\sqrt{1+4p^2R^2}\,. \tag{3.12}$$

The other solution of (3.11) has a leading $u^{-(1/2)-q}$ factor and therefore it is irregular. The 2-point correlator up to a normalization is:

$$\langle\mathcal{O}(p)\mathcal{O}(-p)\rangle = -p^2R^2\left[\psi\left(\frac{3}{2}+q+\sqrt{p^2R^2}\right) + \psi\left(\frac{3}{2}+q-\sqrt{p^2R^2}\right)\right] \tag{3.13}$$

There is a branch cut along the real p^2-axis that extends over the interval $(-1/4R^2, -\infty)$. The spectrum is continuous with a mass gap of $M^2_{gap} = 1/(4R^2) = m^2$ as we observed above. Note that q is purely imaginary after this gap.

Both of the correlation functions (3.10) and (3.13) for large timelike momentums behave as $-p^2\mathrm{lnp}$ which is the pure AdS form.

4. Conclusions

In this section we would like to indicate some open problems and possible applications of the model we studied. This simple system can be a useful toy model for understanding some complicated problems that arise in higher dimensions. For example, when the sigma model manifold is compact, close to the Minkowski vacuum the cosmological constant is positive and gets smaller as we approach to the extremum (figure 1). As mentioned in [25], this

[11] We have used equations [49, (15.1.20), (15.2.1), (15.3.10), (15.3.11)] and some properties of the ψ-function in deriving (3.10) and (3.13).

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might be convenient for studying quintessence scenario [50]. If dS/CFT correspondence [51] is realized, then the holographic duals of euclidean AdS space and dS space may be related to each other [52]. The de Sitter vacua that appears when $\epsilon = 1$, might be used to explore this connection in supergravity.(See also [53] for some comments.)

An important problem is to find the M-theory origin of this model. In fact this is a general problem for most of the known 3-dimensional gauged supergravities because for vector fields there is a Chern-Simons term instead of the usual kinetic term in the lagrangian [10, 14, 15]. A possible solution might be reduction with background fluxes [54].

After finding the compactification, the next step is to identify the CFT dual of this model. To do this one has to figure out the brane configuration that gives rise to that particular near-horizon geometry and then find its world-volume description. This would allow us to compare the results of this paper such as he ratio (2.21) and massless scalar spectrums, with the CFT.

It would be nice to understand the constant a^2 both from the field theory and the M-theory point of view. The naked singularity appeared in the supersymmetric solution when $\epsilon = -1$ is acceptable for $a^2 \leq 1/2$ by the criteria given in [27]. It is reasonable to expect that the dimensional reduction will fix the value of a^2 from this interval. In order to explain why $a^2 > 1/2$ is excluded the solution has to be lifted to M-theory. It may correspond to brane distributions with negative charge and tension [41, 29, 43]. When $\epsilon = -1$, for $a^2 = 1/2$ we found a discrete spectrum with $V_{QM}(z) = -1/(4z^2)$ and because of that $a^2 = 1/2$ may distinguish itself as happened in 5-dimensions [4, 27]. When $\epsilon = 1$, $a^2 = 1/2$ is again special since the black string solution (2.15) coincides with the one found in [55] which is a solution of the 2+1 dimensional string theory [10]. Clearly, nothing much can be said without determining the higher dimensional origin.

The singularity that is present when $\epsilon = -1$ is probably due to some strong coupling effect in the dual CFT [26, 42, 27]. Wilson loops should be calculated to see whether there is a confinement or screening [45] phenomena in the field theory.

A natural continuation of the present work is to consider fluctuations of fields other than massless scalars such as vector field, fermions, active and inert scalars, and apply the holographic renormalization techniques (see [56] for a review and [7, 8] for its use in $D = 3$) to obtain counterterms and correlators. We hope to do these in near future.

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